

2015年1期2日目第4問

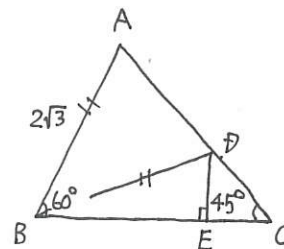


4  $AB = 2\sqrt{3}$ ,  $\angle B = 60^\circ$ ,  $\angle C = 45^\circ$  の三角形 ABC について次の各問の空欄に当てはまる最も適切な数値を記入せよ.

(1)  $AC = \frac{3}{28} \sqrt{\frac{2}{29}}$ ,  $BC = \sqrt{\frac{3}{30}} + \frac{3}{31}$  である.

(2)  $\cos \angle BAC = \frac{\sqrt{\frac{32}{34}} - \sqrt{\frac{33}{34}}}{4}$  である.

(3) 辺 AC 上に  $BA = BD$  を満たす A と異なる点 D を定め, 更に辺 BC 上に  $\angle BED = 90^\circ$  を満たす点 E を定めると,  $AD = \frac{35}{3} \sqrt{\frac{36}{2}} - \sqrt{\frac{37}{6}}$ ,  $BE = \frac{38}{3}$  である.



(1) 正弦定理より,  $\frac{2\sqrt{3}}{\sin 45^\circ} = \frac{AC}{\sin 60^\circ}$

$$\therefore AC = \frac{2\sqrt{3} \cdot \frac{\sqrt{3}}{2}}{\frac{1}{\sqrt{2}}} = 3\sqrt{2} //$$

余弦定理より,  $(3\sqrt{2})^2 = (2\sqrt{3})^2 + BC^2 - 2 \cdot 2\sqrt{3} \cdot BC \cdot \cos 60^\circ$

$$\therefore 18 = 12 + BC^2 - 2\sqrt{3}BC$$

$$\therefore BC^2 - 2\sqrt{3}BC - 6 = 0 \quad \therefore BC = \frac{2\sqrt{3} \pm \sqrt{12 + 4 \cdot 6}}{2} = \sqrt{3} \pm 3$$

$BC > 0$  より,  $BC = \sqrt{3} + 3 //$

(2) 余弦定理より,  $\cos \angle BAC = \frac{(2\sqrt{3})^2 + (3\sqrt{2})^2 - (\sqrt{3} + 3)^2}{2 \cdot 2\sqrt{3} \cdot 3\sqrt{2}} = \frac{\sqrt{6} - \sqrt{2}}{4} //$

(3) 余弦定理より,  $BD^2 = (2\sqrt{3})^2 + AD^2 - 2 \cdot 2\sqrt{3} \cdot AD \cdot \cos \angle BAC$

$BD = BA = 2\sqrt{3}$  より,  $AD^2 - (3\sqrt{2} - \sqrt{6})AD = 0$

$$\therefore AD \{AD - (3\sqrt{2} - \sqrt{6})\} = 0 \quad AD > 0 \text{ より, } AD = 3\sqrt{2} - \sqrt{6} //$$

$$CD = AC - AD = 3\sqrt{2} - (3\sqrt{2} - \sqrt{6}) = \sqrt{6}$$

$$\therefore CE = \frac{CD}{\sqrt{2}} = \sqrt{3} \quad \therefore BE = BC - \sqrt{3} = 3 //$$