

2014年 基幹理工・創造理工・先進理工 第1問

 数理
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 1 複素数 $\alpha = \frac{-1 + \sqrt{3}i}{2}$ に対して,

$$S_n = \sum_{k=1}^n \alpha^{k-1}, \quad T_n = \sum_{k=1}^n k\alpha^{k-1} \quad (n=1, 2, \dots)$$

 とおく. ただし, $\alpha^0 = 1$ とする. 次の問に答えよ.
(1) S_{3m} ($m=1, 2, \dots$) を求めよ.(2) T_{3m} ($m=1, 2, \dots$) を求めよ.(3) T_{2014} を求めよ.

$$(1) \alpha^2 = \frac{-1 - \sqrt{3}i}{2} \text{ より, } \alpha^2 + \alpha + 1 = 0 \dots \textcircled{1} \quad \text{また, } \alpha^3 = 1 \dots \textcircled{2}$$

$$\begin{aligned} \therefore S_{3m} &= 1 + \alpha + \alpha^2 + \alpha^3 + \alpha^4 + \alpha^5 + \alpha^6 + \dots + \alpha^{3m-1} \\ &= (1 + \alpha + \alpha^2) + \alpha^3(1 + \alpha + \alpha^2) + \alpha^6(1 + \alpha + \alpha^2) + \dots + \alpha^{3(m-1)}(1 + \alpha + \alpha^2) \\ &= 0 \quad (\because \textcircled{1} \text{ より}) \\ &\quad \underline{\hspace{2cm} \text{〃}} \end{aligned}$$

$$(2) T_n - \alpha T_n = S_n - n\alpha^n$$

$$\therefore (1 - \alpha)T_n = S_n - n\alpha^n$$

$$(1 - \alpha)T_{3m} = S_{3m} - 3m \cdot \alpha^{3m}$$

$$(1) \text{ より } T_{3m} = \frac{-3m \cdot \alpha^{3m}}{1 - \alpha} = \frac{3m \cdot \alpha^{3m}}{\alpha - 1} = \frac{3m}{\alpha - 1} = \frac{-3 - \sqrt{3}i}{2} m$$

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$$\begin{aligned} T_n &= 1 \cdot \alpha^0 + 2 \cdot \alpha^1 + \dots + n \cdot \alpha^{n-1} \\ \alpha T_n &= 1 \cdot \alpha^1 + \dots + (n-1) \cdot \alpha^{n-1} + n \cdot \alpha^n \\ \hline T_n - \alpha T_n &= 1 + \sum_{k=2}^n \alpha^{k-1} - n\alpha^n \end{aligned}$$

$$(3) T_{2014} = T_{2013} + 2014 \cdot \alpha^{2013}$$

$$= \frac{-3 - \sqrt{3}i}{2} \cdot 671 + 2014 \cdot (\alpha^3)^{671}$$

$$= \frac{-2013 - 671\sqrt{3}i}{2} + 2014$$

$$= \frac{2015 - 671\sqrt{3}i}{2}$$

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