2014年 学芸（情報科学）第2問

数理
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2 次の問に答えよ.

(1) $2\cos\dfrac{2}{5}\pi - 2\cos\dfrac{\pi}{5} + 1 = 0$ が成り立つことを利用して $\cos\dfrac{\pi}{5}$ の値を求めよ.

(2) $\cos\dfrac{\pi}{5}\cdot\cos\dfrac{2}{5}\pi\cdot\cos\dfrac{3}{5}\pi\cdot\cos\dfrac{4}{5}\pi$ の値を求めよ.

(1) $\cos\dfrac{2}{5}\pi = 2\cos^2\dfrac{\pi}{5} - 1$ より.

$4\cos^2\dfrac{\pi}{5} - 2\cos\dfrac{\pi}{5} - 1 = 0$ $\quad \therefore \cos\dfrac{\pi}{5} > 0$ より. $\cos\dfrac{\pi}{5} = \dfrac{1+\sqrt{5}}{4}$

(2) $(与式) = \cos\dfrac{\pi}{5}\cdot\cos\dfrac{2}{5}\pi\cdot\cos\left(\pi-\dfrac{2}{5}\pi\right)\cdot\cos\left(\pi-\dfrac{\pi}{5}\right)$

$= \cos\dfrac{\pi}{5}\cdot\cos\dfrac{2}{5}\pi\cdot\left\{-\cos\dfrac{2}{5}\pi\right\}\cdot\left\{-\cos\dfrac{\pi}{5}\right\}$

$= \cos^2\dfrac{\pi}{5}\cdot\cos^2\dfrac{2}{5}\pi$

$= \cos^2\dfrac{\pi}{5}\cdot\left(\cos\dfrac{\pi}{5} - \dfrac{1}{2}\right)^2$

$= \left(\dfrac{1+\sqrt{5}}{4}\right)^2\cdot\left(\dfrac{-1+\sqrt{5}}{4}\right)^2$

$= \left(\dfrac{4}{16}\right)^2$

$= \dfrac{1}{16}$