



2015年文系第1問

 数理  
石井K

 1 次の性質をもつ数列  $\{a_n\}$  を考える.

$$a_1 = 3$$

$$a_{n+1} > a_n \quad (n = 1, 2, 3, \dots)$$

$$a_n^2 - 2a_n a_{n+1} + a_{n+1}^2 = 3(a_n + a_{n+1}) \quad (n = 1, 2, 3, \dots)$$

(1)  $n = 1, 2, 3, \dots$  に対し,  $a_n + a_{n+2}$  を  $a_{n+1}$  を用いて表せ.(2)  $b_n = a_{n+1} - a_n$  ( $n = 1, 2, 3, \dots$ ) により定まる数列  $\{b_n\}$  の一般項を求めよ.(3) 数列  $\{a_n\}$  の一般項を求めよ.

$$(1) (a_n - a_{n+1})^2 = 3(a_n + a_{n+1}) \quad \dots \textcircled{1}$$

$$(a_{n+1} - a_{n+2})^2 = 3(a_{n+1} + a_{n+2}) \quad \dots \textcircled{2}$$

$$\textcircled{2} - \textcircled{1} \text{ より, } (a_n - a_{n+2})(-a_n + 2a_{n+1} - a_{n+2}) = 3(a_{n+2} - a_n)$$

$$\therefore (a_n - a_{n+2})(-a_n + 2a_{n+1} - a_{n+2} + 3) = 0$$

$$a_{n+2} > a_{n+1} > a_n \text{ より } a_n - a_{n+2} \neq 0 \text{ より, } -a_n + 2a_{n+1} - a_{n+2} + 3 = 0$$

$$\therefore \underline{a_n + a_{n+2} = 2a_{n+1} + 3} //$$

$$(2) (1) \text{ より, } a_{n+2} - a_{n+1} = a_{n+1} - a_n + 3$$

$$\therefore b_{n+1} = b_n + 3$$

 $\therefore$  数列  $\{b_n\}$  は初項  $a_2 - a_1 = 9 - 3 = 6$ , 公差 3 の等差数列

$$\therefore b_n = 6 + 3(n-1) \quad \therefore \underline{b_n = 3n + 3} //$$

$$(3) (2) \text{ より, } a_{n+1} - a_n = 3n + 3$$

$$\therefore a_n = a_1 + \sum_{k=1}^{n-1} (3k + 3) \quad (n \geq 2)$$

$$= 3 + \frac{3}{2}n(n-1) + 3(n-1)$$

$$= \underline{\frac{3}{2}n(n+1)} \quad \text{これは } n=1 \text{ のときも成り立つ}$$