

2016年工学部第4問


 数理
5井

4 定積分

$$I = \int_0^{\frac{\pi}{6}} \frac{\cos x}{\sqrt{3} \sin x + \cos x} dx, \quad J = \int_0^{\frac{\pi}{6}} \frac{\sin x}{\sqrt{3} \sin x + \cos x} dx$$

について以下の問いに答えよ.

- (1) $I + \sqrt{3}J$ の値を求めよ.
 (2) $\sqrt{3}I - J$ の値を求めよ.
 (3) I, J の値を求めよ.

$$(1) I + \sqrt{3}J = \int_0^{\frac{\pi}{6}} \frac{\cos x + \sqrt{3} \sin x}{\sqrt{3} \sin x + \cos x} dx = \int_0^{\frac{\pi}{6}} dx = [x]_0^{\frac{\pi}{6}} = \frac{\pi}{6} "$$

$$\begin{aligned} (2) \sqrt{3}I - J &= \int_0^{\frac{\pi}{6}} \frac{\sqrt{3} \cos x - \sin x}{\sqrt{3} \sin x + \cos x} dx \\ &= \int_0^{\frac{\pi}{6}} \frac{(\sqrt{3} \sin x + \cos x)'}{\sqrt{3} \sin x + \cos x} dx \\ &= \left[\log |\sqrt{3} \sin x + \cos x| \right]_0^{\frac{\pi}{6}} \\ &= \log \sqrt{3} \\ &= \frac{1}{2} \log 3 \end{aligned} "$$

$$(3) (1) \text{より, } I + \sqrt{3}J = \frac{\pi}{6} \dots \textcircled{1}$$

$$(2) \text{より, } \sqrt{3}I - J = \frac{1}{2} \log 3 \dots \textcircled{2}$$

$$\textcircled{1} + \sqrt{3} \times \textcircled{2} \text{より, } 4I = \frac{\pi}{6} + \frac{\sqrt{3}}{2} \log 3$$

$$\therefore I = \frac{\pi}{24} + \frac{\sqrt{3}}{8} \log 3 "$$

$$\sqrt{3} \times \textcircled{1} - \textcircled{2} \text{より, } 4J = \frac{\sqrt{3}}{6} \pi - \frac{1}{2} \log 3$$

$$\therefore J = \frac{\sqrt{3}}{24} \pi - \frac{1}{8} \log 3 "$$