

2016年医学部第9問

9 複素数 z は, $1 + z + z^2 + z^3 + z^4 + z^5 + z^6 + z^7 + z^8 + z^9 = 0$ を満たす. $\frac{|z-2|^2 + |z+2|^2}{5}$ の値を求めよ.

$$\begin{aligned}(\text{手式}) &= \frac{(z-2)(\bar{z}-2) + (z+2)(\bar{z}+2)}{5} \\ &= \frac{2|z|^2 + 8}{5}\end{aligned}$$

ここで, $(1-z)(1+z+z^2+\dots+z^9) = 1-z^{10}$ より

$$z^{10} = 1 \quad \therefore |z| = 1$$

$$\therefore (\text{手式}) = \frac{2+8}{5} = 2 //$$