自治医科大学

2011年第6問

数理 石井K

$\boxed{6}$ x，yは，$0 < x < \frac{1}{3}$，$0 < y < \frac{1}{3}$ を満たす実数とする．$x = b$，$y = c$のとき，

$$\log_x y + \log_y x = 2$$
$$2\log_x \sin\{\pi(x+y)\} = \log_x \sin(\pi y) + \log_y \cos(\pi x)$$

を満たす．$12b$の値を求めよ．

$\log_b C + \log_c b = 2$ より $\log_b C + \dfrac{\log_b b}{\log_b C} = 2$

$\therefore$ $t = \log_b C$ とおくと，$t^2 - 2t + 1 = 0$

$(t-1)^2 = 0$ $\therefore$ $t = 1$ $\therefore$ $b = C$

これより，$2\log_b \sin 2b\pi = \log_b \sin \pi \cdot b + \log_b \cos \pi b$

$\therefore$ $\log_b (\sin 2b\pi)^2 = \log_b \sin b\pi \cos b\pi$

$\log_b \dfrac{(\sin 2b\pi)^2}{\frac{1}{2}\sin 2b\pi} = 0$

$\therefore$ $2\sin 2b\pi = 1$

$\sin 2b\pi = \dfrac{1}{2}$

$0 < b < \frac{1}{3}$ より，$0 < 2b\pi < \frac{2}{3}\pi$ なので $2b\pi = \dfrac{\pi}{6}$

$\therefore$ $2b = \dfrac{1}{6}$ $b = \dfrac{1}{12}$

$12b = \underline{1}$ //