

2015年A日程第1問

 数理  
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 1  $a = 1 + \sqrt{6} + \sqrt{7}$ ,  $b = 1 - \sqrt{6} + \sqrt{7}$  のとき, 次の値を求めよ.

$$(1) ab = \boxed{\phantom{000}} \quad 2 + 2\sqrt{7}$$

$$(2) \frac{1}{a} = \boxed{\phantom{000}} \quad \frac{6 + \sqrt{6} - \sqrt{42}}{12}$$

$$(3) \frac{1}{a} + \frac{1}{b} = \boxed{\phantom{000}} \quad 1$$

$$(4) \frac{a}{b} - \frac{b}{a} = \boxed{\phantom{000}} \quad 2\sqrt{6}$$

$$\begin{aligned} (1) ab &= \{(1 + \sqrt{7}) + \sqrt{6}\} \{(1 + \sqrt{7}) - \sqrt{6}\} \\ &= (1 + \sqrt{7})^2 - 6 \\ &= \underline{2 + 2\sqrt{7}} \quad \text{,,} \end{aligned}$$

$$\begin{aligned} (2) \frac{1}{a} &= \frac{b}{ab} \\ &= \frac{1 - \sqrt{6} + \sqrt{7}}{2 + 2\sqrt{7}} \\ &= \frac{(\sqrt{7} - 1)(1 - \sqrt{6} + \sqrt{7})}{2(\sqrt{7} + 1)(\sqrt{7} - 1)} \\ &= \frac{\sqrt{7} - \sqrt{42} + 7 - 1 + \sqrt{6} - \sqrt{7}}{2(7 - 1)} \\ &= \frac{6 + \sqrt{6} - \sqrt{42}}{12} \quad \text{,,} \end{aligned}$$

$$\begin{aligned} (3) a + b &= 1 + \sqrt{6} + \sqrt{7} + 1 - \sqrt{6} + \sqrt{7} \\ &= 2 + 2\sqrt{7} \end{aligned}$$

$$\therefore \frac{1}{a} + \frac{1}{b} = \frac{a+b}{ab} = \frac{2+2\sqrt{7}}{2+2\sqrt{7}} = \underline{1} \quad \text{,,}$$

$$\begin{aligned} (4) \frac{a}{b} - \frac{b}{a} &= \frac{a^2 - b^2}{ab} \\ &= \frac{(a+b)(a-b)}{ab} \end{aligned}$$

$$\begin{aligned} \therefore \because a - b &= 1 + \sqrt{6} + \sqrt{7} - (1 - \sqrt{6} + \sqrt{7}) \\ &= 2\sqrt{6} \end{aligned}$$

$$\begin{aligned} \therefore \frac{a}{b} - \frac{b}{a} &= \frac{(2+2\sqrt{7}) \cdot 2\sqrt{6}}{2+2\sqrt{7}} \\ &= \underline{2\sqrt{6}} \quad \text{,,} \end{aligned}$$