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名城大学

2014年 理工学部 第4問

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$\boxed{4}$ 数列 $\{a_n\}$ を

$$a_n = \int_0^1 |nx-1|e^x\,dx \quad (n=1,\ 2,\ 3,\ \cdots)$$

で定める．次の各問に答えよ．

(1) a_1, a_2 を求めよ．

(2) a_n を求めよ．

(3) $\displaystyle\lim_{n\to\infty}\frac{a_n}{n}$ を求めよ．

(1)
$$\begin{aligned}
a_1 &= \int_0^1 |x-1|e^x\,dx \\
&= \int_0^1 (1-x)e^x\,dx \\
&= \int_0^1 (1-x)(e^x)'\,dx \\
&= \left[(1-x)e^x\right]_0^1 - \int_0^1 -e^x\,dx \\
&= -1 + \left[e^x\right]_0^1 \\
&= \underline{e-2}
\end{aligned}$$

$$\begin{aligned}
a_2 &= \int_0^1 |2x-1|e^x\,dx \\
&= \int_0^{\frac{1}{2}} (1-2x)e^x\,dx + \int_{\frac{1}{2}}^1 (2x-1)e^x\,dx \\
&= \left[(1-2x)e^x\right]_0^{\frac{1}{2}} - \int_0^{\frac{1}{2}} -2e^x\,dx \\
&\quad + \left[(2x-1)e^x\right]_{\frac{1}{2}}^1 - \int_{\frac{1}{2}}^1 2e^x\,dx \\
&= -1 + \left[2e^x\right]_0^{\frac{1}{2}} + e - \left[2e^x\right]_{\frac{1}{2}}^1 \\
&= -1 + 2\sqrt{e} - 2 + e - 2e + 2\sqrt{e} \\
&= \underline{-e+4\sqrt{e}-3}
\end{aligned}$$

(2)

$x \geqq \frac{1}{n}$ で $nx-1 \geqq 0$

$$\begin{aligned}
a_n &= \int_0^1 |nx-1|e^x\,dx \\
&= \int_0^{\frac{1}{n}} (1-nx)e^x\,dx + \int_{\frac{1}{n}}^1 (nx-1)e^x\,dx \\
&= \left[(1-nx)e^x\right]_0^{\frac{1}{n}} - \int_0^{\frac{1}{n}} -ne^x\,dx \\
&\quad + \left[(nx-1)e^x\right]_{\frac{1}{n}}^1 - \int_{\frac{1}{n}}^1 ne^x\,dx \\
&= -1 + \left[ne^x\right]_0^{\frac{1}{n}} + (n-1)e - \left[ne^x\right]_{\frac{1}{n}}^1 \\
&= -1 + ne^{\frac{1}{n}} - n + (n-1)e - ne + ne^{\frac{1}{n}} \\
&= \underline{2ne^{\frac{1}{n}} - n - e - 1}
\end{aligned}$$

(3) (2) より．

$$\begin{aligned}
\lim_{n\to\infty}\frac{a_n}{n} &= \lim_{n\to\infty} \underbrace{2e^{\frac{1}{n}}}_{\to 2} - 1 - \frac{e}{n} - \frac{1}{n} \\
&= \underline{1}
\end{aligned}$$