



2014年第1問



1 平面上のベクトル

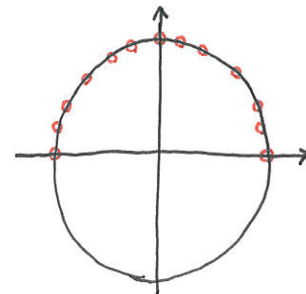
$$\vec{a}_n = \left(\cos \frac{n\pi}{4}, \sin \frac{n\pi}{4} \right), \quad \vec{b}_n = \left(2 \cos \frac{n\pi}{6}, 2 \sin \frac{n\pi}{6} \right) \quad (n = 0, 1, 2, \dots, 12)$$

に対して, $\sum_{n=0}^{12} |\vec{a}_n + \vec{b}_n|^2$ を求めよ.

$$\begin{aligned} |\vec{a}_n| &= 1, \quad |\vec{b}_n| = 2, \quad \vec{a}_n \cdot \vec{b}_n = 2 \cos \frac{n\pi}{4} \cos \frac{n\pi}{6} + 2 \sin \frac{n\pi}{4} \sin \frac{n\pi}{6} \\ &= 2 \cos \left(\frac{n\pi}{4} - \frac{n\pi}{6} \right) \\ &= 2 \cos \frac{n\pi}{12} \end{aligned}$$

$$\begin{aligned} \therefore |\vec{a}_n + \vec{b}_n|^2 &= |\vec{a}_n|^2 + 2\vec{a}_n \cdot \vec{b}_n + |\vec{b}_n|^2 \\ &= 5 + 4 \cos \frac{n\pi}{12} \end{aligned}$$

$$\begin{aligned} \sum_{n=0}^{12} |\vec{a}_n + \vec{b}_n|^2 &= \sum_{n=0}^{12} \left(5 + 4 \cos \frac{n\pi}{12} \right) \\ &= 65 + 4 \sum_{n=0}^{12} \cos \frac{n\pi}{12} \end{aligned}$$



$$\begin{aligned} \because \cos 0 + \cos \pi = 0, \quad \cos \frac{\pi}{12} + \cos \frac{11\pi}{12} = 0, \quad \cos \frac{2\pi}{12} + \cos \frac{10\pi}{12} = 0 \\ \cos \frac{3\pi}{12} + \cos \frac{9\pi}{12} = 0, \quad \cos \frac{4\pi}{12} + \cos \frac{8\pi}{12} = 0, \quad \cos \frac{5\pi}{12} + \cos \frac{7\pi}{12} = 0 \end{aligned}$$

であるから,

$$\begin{aligned} \sum_{n=0}^{12} |\vec{a}_n + \vec{b}_n|^2 &= 65 + 4 \cdot \cos \frac{6\pi}{12} \\ &= \underline{\underline{65}} \end{aligned}$$