

2012年 第5問

5 等式  $f(x) = x^2 + x \int_0^1 f(t) dt + \int_{-1}^1 f(t) dt$  を満たす関数  $f(x)$  を求めよ.

$$a = \int_0^1 f(t) dt, \quad b = \int_{-1}^1 f(t) dt \quad \text{とおくと.} \quad f(x) = x^2 + ax + b \quad \text{と表せる}$$

$$\text{よって. } a = \int_0^1 t^2 + at + b \, dt$$

$$= \left[ \frac{t^3}{3} + \frac{a}{2} t^2 + bt \right]_0^1$$

$$= \frac{1}{3} + \frac{a}{2} + b$$

$$\therefore \frac{a}{2} - b = \frac{1}{3} \quad \Leftrightarrow \quad a - 2b = \frac{2}{3} \quad \cdots \textcircled{1}$$

また,

$$b = \int_{-1}^1 t^2 + at + b \, dt$$

$$= \left[ \frac{t^3}{3} + \frac{a}{2} t^2 + bt \right]_{-1}^1$$

$$= \frac{2}{3} + 2b$$

$$\therefore b = -\frac{2}{3} \quad \cdots \textcircled{2}$$

$$\textcircled{1}, \textcircled{2} \text{ より. } \quad a = -\frac{2}{3}, \quad b = -\frac{2}{3}$$

$$\therefore \underline{f(x) = x^2 - \frac{2}{3}x - \frac{2}{3}} \quad "$$