兵庫県立大学

2015年 工学部 第3問

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3 次の定積分の値を求めよ.

(1) $\displaystyle\int_0^{\frac{\pi}{3}} \frac{dx}{\cos^2 x}$

(2) $\displaystyle\int_0^{\frac{\pi}{3}} \frac{dx}{\cos^4 x}$

(1) $\displaystyle\int_0^{\frac{\pi}{3}} \frac{dx}{\cos^2 x} = \left[\tan x\right]_0^{\frac{\pi}{3}}$

$= \underline{\sqrt{3}}$ //

(2) $I = \displaystyle\int_0^{\frac{\pi}{3}} \frac{dx}{\cos^4 x}$ とおくと,

$I = \displaystyle\int_0^{\frac{\pi}{3}} \frac{1}{\cos^2 x} \cdot \frac{1}{\cos^2 x}\, dx$

$= \displaystyle\int_0^{\frac{\pi}{3}} (\tan x)' \cdot \frac{1}{\cos^2 x}\, dx$

$= \left[\dfrac{\tan x}{\cos^2 x}\right]_0^{\frac{\pi}{3}} - \displaystyle\int_0^{\frac{\pi}{3}} \tan x \cdot \frac{2\sin x \cos x}{\cos^4 x}\, dx$

$= \dfrac{\sqrt{3}}{\left(\frac{1}{2}\right)^2} - 2\displaystyle\int_0^{\frac{\pi}{3}} \frac{\sin^2 x}{\cos^4 x}\, dx$

$= 4\sqrt{3} - 2\displaystyle\int_0^{\frac{\pi}{3}} \frac{1-\cos^2 x}{\cos^4 x}\, dx$

$= 4\sqrt{3} - 2I + 2\displaystyle\int_0^{\frac{\pi}{3}} \frac{dx}{\cos^2 x}$

$= 4\sqrt{3} - 2I + 2\sqrt{3}$ $(\because$ (1) より$)$

よって, $3I = 6\sqrt{3}$ $\therefore \underline{I = 2\sqrt{3}}$ //