2014年 第5問

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$\boxed{5}$ a を正の数とする．このとき，次の関係式をみたす関数 $f(x)$ を求めよ．

$$f(x) = \int_0^{\frac{\pi}{a}} f(t)\cos(at - 2ax)\,dt + 1$$

$$f(x) = \int_0^{\frac{\pi}{a}} f(t)\left(\cos at \cos 2ax + \sin at \sin 2ax\right) dt + 1$$

$$= \cos 2ax \int_0^{\frac{\pi}{a}} f(t)\cos at\ dt + \sin 2ax \int_0^{\frac{\pi}{a}} f(t)\sin at\ dt + 1$$

ここで，$p = \int_0^{\frac{\pi}{a}} f(t)\cos at\ dt$， $q = \int_0^{\frac{\pi}{a}} f(t)\sin at\ dt$ とおくと，

$f(x) = p\cos 2ax + q\sin 2ax + 1 \quad \cdots ①$ と表せる

$$\therefore\ p = \int_0^{\frac{\pi}{a}} (p\cos 2at + q\sin 2at + 1)\cos at\ dt$$

$$= \int_0^{\frac{\pi}{a}} p\cos 2at\cos at + q\sin 2at\cos at + \cos at\ dt$$

積・和の公式より．

$$= \int_0^{\frac{\pi}{a}} \frac{p}{2}(\cos 3at + \cos at) + \frac{q}{2}(\sin 3at + \sin at) + \cos at\ dt$$

$$= \left[\frac{p}{2}\left(\frac{1}{3a}\sin 3at + \frac{1}{a}\sin at\right)\right]_0^{\frac{\pi}{a}} + \left[\frac{q}{2}\left(-\frac{1}{3a}\cos 3at - \frac{1}{a}\cos at\right)\right]_0^{\frac{\pi}{a}} + \left[\frac{1}{a}\sin at\right]_0^{\frac{\pi}{a}}$$

$$= \frac{q}{2}\left(\frac{1}{3a} + \frac{1}{a} + \frac{1}{3a} + \frac{1}{a}\right)$$

$$= \frac{4q}{3a}$$

$$\therefore\ p = \frac{4q}{3a} \quad \cdots ②$$

また，$q = \int_0^{\frac{\pi}{a}} (p\cos 2at + q\sin 2at + 1)\sin at\ dt$

$$= \int_0^{\frac{\pi}{a}} p\sin at\cos 2at + q\sin 2at\sin at + \sin at\ dt$$

積・和の公式より．

$$= \int_0^{\frac{\pi}{a}} \frac{p}{2}(\sin 3at - \sin at) + \frac{q}{2}(\cos 3at - \cos at) + \sin at\ dt$$

$$= \left[\frac{p}{2}\left(-\frac{1}{3a}\cos 3at + \frac{1}{a}\cos at\right)\right]_0^{\frac{\pi}{a}} + \left[\frac{q}{2}\left(\frac{1}{3a}\sin 3at - \frac{1}{a}\sin at\right)\right]_0^{\frac{\pi}{a}} + \left[-\frac{1}{a}\cos at\right]_0^{\frac{\pi}{a}}$$

$$= \frac{p}{2}\left(\frac{1}{3a} - \frac{1}{a} + \frac{1}{3a} - \frac{1}{a}\right) + \frac{1}{a} + \frac{1}{a}$$

$$\therefore\ q = -\frac{2p}{3a} + \frac{2}{a} \quad \cdots ③$$

②，③より，$p = \dfrac{24}{9a^2+8}$，$q = \dfrac{18a}{9a^2+8}$　①に代入して，$\underline{f(x) = \dfrac{24}{9a^2+8}\cos 2ax + \dfrac{18a}{9a^2+8}\sin 2ax + 1}$ //