

2015年文系第4問



4 $0^\circ \leq \theta \leq 45^\circ$ で $\sin \theta \cos \theta = \frac{2}{5}$ のとき、次の式の値をそれぞれ求めなさい。

(1) $(\sin \theta + \cos \theta)^2$

(2) $\sin \theta + \cos \theta$

(3) $(\sin \theta - \cos \theta)^2$

(4) $\sin \theta - \cos \theta$

(5) $\sin \theta$

$$\begin{aligned}
 (1) (\sin \theta + \cos \theta)^2 &= \sin^2 \theta + \cos^2 \theta + 2 \sin \theta \cos \theta \\
 &= 1 + \frac{4}{5} \\
 &= \frac{9}{5} \text{ 〃}
 \end{aligned}$$

(2) $0^\circ \leq \theta \leq 45^\circ$ より、 $\sin \theta + \cos \theta > 0$

$$\therefore (1) \text{ より、} \sin \theta + \cos \theta = \frac{3\sqrt{5}}{5} \text{ 〃}$$

$$\begin{aligned}
 (3) (\sin \theta - \cos \theta)^2 &= \sin^2 \theta + \cos^2 \theta - 2 \sin \theta \cos \theta \\
 &= 1 - \frac{4}{5} \\
 &= \frac{1}{5} \text{ 〃}
 \end{aligned}$$

(4) $0^\circ \leq \theta \leq 45^\circ$ より、 $\sin \theta \leq \cos \theta$ $\therefore \sin \theta - \cos \theta \leq 0$

$$\therefore (3) \text{ より、} \sin \theta - \cos \theta = -\frac{\sqrt{5}}{5} \text{ 〃}$$

(5) $\sin \theta + \cos \theta = \frac{3\sqrt{5}}{5}$... ①

$\sin \theta - \cos \theta = -\frac{\sqrt{5}}{5}$... ②

$$\text{①} + \text{②} \text{ より、} 2 \sin \theta = \frac{2\sqrt{5}}{5} \quad \therefore \sin \theta = \frac{\sqrt{5}}{5} \text{ 〃}$$