

2014年第3問

3 次の定積分を求めよ.

(1) $\int_0^2 |e^x - e| dx$

(2) $\int_1^e \frac{\log x}{x^2} dx$

(1) e^x は単調増加で $e^x = e$ と交るのは $x=1$ のとき.

$$\begin{aligned}
 \therefore (1) \text{ (答)} &= \int_0^1 e - e^x dx + \int_1^2 e^x - e dx \\
 &= [ex - e^x]_0^1 + [e^x - ex]_1^2 \\
 &= e - e - (0 - 1) + e^2 - 2e - (e - e) \\
 &= \underline{(e-1)^2}
 \end{aligned}$$

$$\begin{aligned}
 (2) \text{ (答)} &= \int_0^1 e^{-t} \cdot t dt \\
 &= \int_0^1 t \cdot (-e^{-t})' dt
 \end{aligned}$$

$$\begin{aligned}
 t = \log x \text{ とおくと } x = e^t \\
 dt = \frac{1}{x} \cdot dx
 \end{aligned}$$

x	$1 \rightarrow e$
t	$0 \rightarrow 1$

$$= [-te^{-t}]_0^1 - \int_0^1 -e^{-t} dt$$

$$= -e^{-1} + [-e^{-t}]_0^1$$

$$= -\frac{1}{e} + \left(-\frac{1}{e} + 1\right)$$

$$= \underline{1 - \frac{2}{e}}$$